Derivative Pricing and Stochastic Calculus

Number of hours: 48h

First part: 24h

Instructor: Luciano CAMPI
Title/Current position: Assistant Professor, CEREMADE
Mini-biography: Luciano obtained a PhD in applied mathematics at University Paris VI. His research area concerns stochastic calculus applied finance, markets with frictions, insider trading, and energy derivatives. He is a member of the CEREMADE, Paris Dauphine University.

Course objectives:
The purpose of this course is to present the basics of probability theory and continuous time processes, allowing for a rigorous understanding of the standard dynamic stock models. Due to the motivations of the course, the mathematical concepts will be taught, considering concrete financial applications.
After a review of basic probability notions, the course will focus on the rigorous mathematical definition of arbitrage opportunities, leading to basic financial derivative valuation techniques. After a study of option pricing in multiperiod binomial models, we introduce stochastic calculus tools such as Brownian motion, stochastic integral and Ito formula. With these instruments, a complete study of the classical Black Scholes model will be performed: option valuation, PDE price characterization and Greeks computation. If time permits, the last chapter will deal with Monte Carlo simulations and practical calibration and estimation methods for the coefficients of the Black Scholes model.

Course outline:
1. Basics on probability
   Random variable, expectation, variance, law, density, Gaussian law, …
2. Arbitrage
   No arbitrage opportunity conditions, Call Put parity, American call, Forward contracts.
3. A toy example: the Binomial pricing model
   Model dynamics, Risk neutral pricing, Market completeness.
4. Introducing dynamical strategies: multi-period binomial models
   Portfolio strategies, asymptotic behavior with the number of periods
5. Modelling the market randomness: the Brownian motion
   Characterizations, quadratic variation,
   Geometric Brownian Motion, Merton vs. Bachelier
6. Introducing portfolio dynamics: stochastic integration and Ito formula
   Ito calculus, continuous trading strategies, stochastic differential equations
7. The reference in continuous time: Black & Scholes
   Model dynamics, Risk neutral pricing, Greeks computation
   The price as the solution of a partial differential equation
   Volatility calibration and estimation.
   Monte Carlo methods vs. Numerical PDE approximation.

References:
Lamberton, Lapeyre – Introduction to Stochastic Calculus applied to finance.

Second part: 24h

Instructor : Bruno BOUCHARD
Title/Current position: Professor, Université Paris-Dauphine
Mini-biography : Bruno Bouchard is Professor at University Paris-Dauphine and invited Professor at the ENSAE, where he teaches mathematical finance. He is member of the CREST and CEREMADE laboratories, as well as the Europlace Finance Institute. Furthermore, he is associate editor for the journal : *Finance and Stochastics*.

Course objectives:

The aim of this lecture is to present the theory of derivative asset pricing as well as the main models and techniques used in practice. The lecture starts with discrete time models which can be viewed as a proxy for continuous settings. We then develop on the theory of continuous time models. We start with a general Itô-type framework and then specialize to different situations: Markovian models, constant volatility models, local and stochastic volatility models. For each of them, we discuss their calibration, and the valuation and the hedging of different type of options (plain Vanilla and barrier options, contracts on future, American options, options on foreign markets, options on realized variance,...).

Course outline:

0 - Introduction

Introduction to derivatives markets and products.

I – Discrete time modelling

1- Financial assets
   a– General setting
   b- Tree markets
2- The absence of arbitrage
3- Pricing and hedging of European options
   a- The super-hedging problem
   b- The complete market case : example of the CRR model
   c- Approximate hedging in incomplete markets and selection of a pricing measure (example of the trinomial tree model)
4- Pricing and hedging of American options
5- Discrete time models as a proxy for continuous time models
6- Introduction and discussion of some imperfections on the market.

II- Continuous time modelling

1- Financial assets as Itô processes
   i- Discussion of the Absence of arbitrage opportunity
   ii- Complete and incomplete markets
   iii- The general pricing and hedging principle for European and American claims

- iv- Impact of the dividends and non-constant interest rates (forward risk neutral measure)

2- Markovian models in complete markets
   - i- PDE valuation (plain vanilla, barrier, Asian, American options)
   - ii- Greeks and hedging
   - iii- Tracking error and convexity

3- Model studies
   - a- Closed form prices in the Black and Scholes type model
     - i- Plain Vanilla options (examples)
     - ii- Barrier options
     - iii- Futures in the Black model
     - iv- Options on foreign markets and currency derivatives
   - b- Merton's model
   - c- Local volatily models
     - i- Dupire Formula and calibration
     - v- CEV model and FFT valuation method
   - d- Stochastic volatility
     - i- Completion of the market with options : general principal
     - ii- Approximate static hedging : example of the variance swap hedging problem
     - iii- The Heston model
     - iv- SABR model
     - v- Jumps in the volatility (Bates model)
     - vi- Variance swap market models and options on realized variance

References:

- Cont R. et P. Tankov, *Financial Modelling with Jump Processes*, Chapman and Hall, 2004.
- Hull J., *Options, Futures and other Derivatives Securities*, Prentic-Hall, 2002.
- Lamberton D. et B. Lapeyre, *Introduction au calcul stochastique appliqué à la finance*, Ellipses, Paris, 1999.
- Musiela M. et M. Rutkowski, *Martingale methods in financial modelling*, Springer, 1997.

Assessment: 1 final exam