

TERM STRUCTURES: COMMODITIES, INTEREST RATES, AND OTHER ASSETS

D. LAUTIER

OVERVIEW:

The term structure is defined as the relationship between the spot price and the futures prices of a derivative instrument, for any delivery date. It provides useful information for hedging, arbitrage, investment and evaluation: it indeed synthesizes the information available in the market and the operators' expectations concerning the future price of the underlying asset.

In many derivative markets, especially in interest rates and in commodity markets, the concept of term structure is very important, because the contract's maturity increases as the markets come to fruition. In the American crude oil market, for example, there are futures contracts for maturities as far as 9 years; in the Eurodollar market, the maturities reach 10 years.

The first chapter of the course is devoted to a general introduction to derivatives today. The second and third are devoted to the theoretical analysis of the term structures. The second chapter devotes itself primarily to the traditional theories of commodity prices and to the explanation of the relationship between spot and futures prices. An empirical review of the results obtained through these frameworks is presented. These theories are still very important and extensively investigated.

The theories of hedging pressure and storage are however a bit limited when the whole term structure is taken into account. As a result, there is a need for a long-term extension of the analysis, which is the very subject of the third chapter. A dynamic analysis of the term structure is presented. Then the focus turns towards term structure models of commodity prices. These models differ on the nature and the number of factors used to describe uncertainty. They also differ in the valuation method they rely on.

The fourth chapter reviews the main empirical results obtained with term structure models. Simulations highlight the influence of the assumptions concerning the stochastic process retained for the state variables and the number of state variables. Then, the econometric method usually employed for the estimation of the parameters is explained. In such a context, as the variables are non-observable, there is a need for filtering techniques. Finally two main applications are studied, i.e. dynamic hedging and investment valuation.

The last chapter is devoted to the study of structural models, ie micro-founded equilibrium models that also examine the interactions between the physical and the derivative markets. In this situation the spot price becomes endogenous. The interactions between prices are studied thanks to rational expectations equilibriums.

COURSE OBJECTIVES:

At the end of this course, the students must have a broad knowledge about the term structures of derivative prices: the theories, the valuation methods, the econometric techniques, the empirical tests as well as the applications.

MODE OF ASSESSMENT:

One final exam, 100%.

COURSE SCHEDULE:

Chapter 1. Derivative markets and prices: an introduction

Chapter 2. First market models: the theories of normal backwardation and storage

Chapter 3. Term structures: dynamic behaviour, theories and models

Chapter 4. Applications of term structure models

Chapter 5. Structural models of commodity markets

MAIN REFERENCES¹

- Danthine J.P., Donaldson J.B., *Intermediate Financial Theory*, 2d Ed., Elsevier, 2005.
- Hull J., *Options, futures and other derivatives*, 9th Ed.
- Kolb R.W. , Overdahl J.A. , *Futures, options, and swaps*, 5th Ed., Blackwell, 2007.
- Williams J., *The economic function of futures markets*, Cambridge University Press, 1986
- Wilmott P., *Paul Wilmott on Quantitative Finance*, 3-volume set, 2nd Ed., Wiley, 2006.

¹ For each chapter, a more precise bibliography will be provided.

CHAPTER 1. DERIVATIVE MARKETS AND PRICES: AN INTRODUCTION

Section 1. A few definitions

Section 2. The economic functions of derivative markets

Section 3. An overview of derivative markets today

CHAPTER 2. THE FIRST MARKET MODELS

Section 1. The Hedging pressure theory (Normal backwardation)

1.1. Introduction to the theory

1.2. The analysis of the risk premium towards the theory of financial markets

1.3. Empirical tests

1.4. Comparison with other financial assets

Section 2. The storage theory

2.1. The role of inventories in commodity markets

2.2. The analysis of contango and backwardation

2.3. The convenience yield

2.4. Empirical tests of the storage theory

2.5. Critiques of the theory

2.6. Comparison with other financial assets

CHAPTER 3. TERM STRUCTURES: DYNAMIC BEHAVIOUR, THEORIES AND MODELS
--

Section 1. Dynamic behaviour of the term structures

- 1.1. Illustrations
- 1.2. The Samuelson effect
- 1.3. Statistical methods used to describe the movement of prices curves
- 1.3 Dynamic behaviour of the prices curves: illustrations
- 1.4. From curves to surfaces: the third dimension

Section 2. Theories of the term structure

- 2.1. Term structures and institutional factors
- 2.2. Term structures and expectations
- 2.3. Risk and liquidity premiums
- 2.4. Preferred habitat theory
- 2.5. From interest rates to other assets

Section 3. Term structure models

- 3.1. Contingent claim analysis: a brief reminder
- 3.2. One-factor models
- 3.3. Two-factor models
- 3.4. Three-factor models
- 3.5. The HJM framework

Section 4. Empirical validation of term structure models

- 4.1. Simulations
- 4.2. Parameters estimations and the Kalman filters
- 4.3. Performances of the models

CHAPTER 4. APPLICATIONS OF TERM STRUCTURE MODELS
--

Section 1. Dynamic hedging

- 1.1. An introduction to dynamic hedging
- 1.2. An example of dynamic hedging: Metallgesellschaft
- 1.3. Term structure models and dynamic hedging strategies
 - 1.3.1. The hedging problem
 - 1.3.2. Arbitrage reasoning and hedge portfolios
 - 1.3.3. Hedging positions
 - 1.3.4. Analysis of the hedging errors
 - 1.3.5. Empirical tests
 - 1.3.6. Conclusions

Section 2. Options valuation and investment

- 2.1. The real options theory
- 2.2. Application to commodities
- 2.3. Case study

CHAPTER 5. STRUCTURAL MODELS OF COMMODITY PRICES
--

Section 1. Structural models and traditional theories

- 1.1. Economic setting
 - 1.1.1 Fundamental economic structures
 - 1.1.2 The operators
- 1.2 Micro-economics foundations
 - 1.2.1. Utility functions: assumption
 - 1.2.2. Maximization in the general framework
 - 1.2.3. Mean variance framework
- 1.3 Equilibrium analysis
 - 1.3.1 Rational expectation equilibrium: reminders
 - 1.3.2 Solving the equilibrium equations
 - 1.3.3 Three different cases of equilibrium

Section 2. Multi-period setting

- 2.1. Analysis of an individual hedger
- 2.2. Hedging price and quantity risk
- 2.3. Futures market equilibrium
 - 2.3.1. Resolution of uncertainty over time
 - 2.3.2. Description of the model
 - 2.3.3. A first restriction
 - 2.3.4. A further restriction