

Les objectifs du cours

The aim of this lecture is to present the theory of derivative asset pricing as well as the main models and techniques used in practice. The lecture starts with discrete time models which can be viewed as a proxy for continuous settings. We then develop on the theory of continuous time models. We start with a general Itô-type framework and then specialize to different situations: Markovian models, constant volatility models, local and stochastic volatility models. For each of them, we discuss their calibration, and the valuation and the hedging of different types of options (plain Vanilla and barrier options, American options, options on realized variance,...).

Plan du cours

I. Discrete time modelling

I.1. Financial assets

- General setting
- Tree markets

I.2. The absence of arbitrage

- Risk-neutral measures
- Fundamental theorem of asset pricing

I.3. Pricing and hedging of European options

- The super-hedging problem
- The complete market case
- Approximate hedging in incomplete markets
- Examples : binomial and trinomial tree markets

I.4. Pricing and hedging of American options

II. Continuous time modelling

II.1. Financial assets as Itô processes

- The Itô process framework
- Discussion of the Absence of arbitrage opportunity
- Complete and incomplete markets
- The general pricing and hedging principle for European and American claims

II.2 The Black-Scholes model

- Characterization of complete Black Scholes markets
- Explicit formulas : European call option (Black-Scholes formula), barrier option (reflection principle)

III.2. Markovian models in complete markets

- PDE valuation (plain vanilla, barrier, Asian, American options)
- Greeks and hedging
- Tracking error and convexity

III.3 Local volatility models

- Dupire's formula and calibration to the volatility surface

II.4 Stochastic volatility models

- Super hedging prices
- Completion of the market with options : general principle, Approximate static hedging : example of the variance swap hedging problem
- Specific models : CEV, Heston, SABR,...

Bibliographie

Bouchard B. et Chassagneux J.F., Fundamentals and advanced Techniques in derivatives hedging, Springer, 2016.

Lamberton D. et B. Lapeyre, Introduction au calcul stochastique appliqué à la finance, Ellipses, Paris, 1999.
